Curves and Surfaces Mid Term Exam

February 20 2025

This exam is of **30 marks** and is **3 hours long** - from 10 am to 1pm. Please **read all the questions carefully**. Please feel free to use whatever theorems you have learned in class after stating them clearly.

1, Recall that the **Normal Plane** at a point P is the plane spanned by $\mathbf{N}((P)$ and $\mathbf{B}(P)$. Prove that if all normal planes of a curve pass through a particular point then the curve lies on a sphere. (5)

2. Calculate the Frenet frame $\mathbf{T}, \mathbf{N}, \mathbf{B}$, the curvature κ and the torsion τ for the following curve (10)

$$\alpha(t) = \left(t, t^2/2, t\sqrt{1+t^2} + \log(t + \sqrt{1+t^2})\right)$$

3. Compute the following for the surface given by the parametrization

$$x(u,v) -= (a + b\cos(u)\cos(v), (a + b\cos(u))\sin(v), b\sin(u))\ (0 < b < a)$$

- I_P (2)
- II_P (2)
- Matrix of the shape operator S_P (2)
- The surface area in the region $0 \leq u, v \leq 2\pi$ (5)
- The mean curvature H (2)
- The Gaussian curvature K. (2)

4. Prove or give a counterexample: If M is a surface with Gaussian curvature $K > 0$ then the curvature of any curve $C \subset M$ is everywhere positive. (Recall that by definition the curvature of any curve $\kappa_C \geq 0$ always) (5)